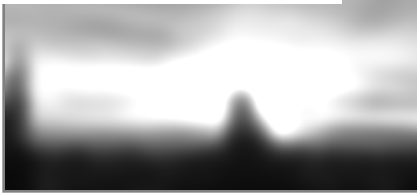


Bilateral Filtering for Tone Mapping

Chiu et al. 1993

- › Reduce contrast of low-frequencies
- › Keep high frequencies

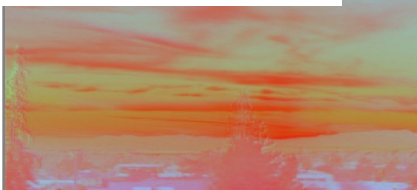
Low-freq.



High-freq.



Color



Reduce low frequency

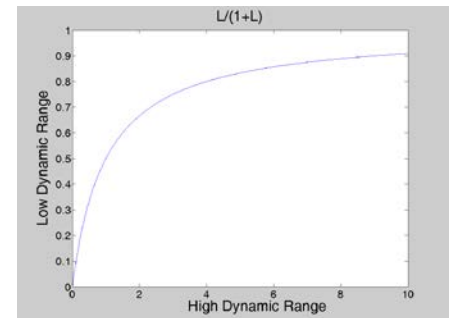


How Does It Work?

› Local tone mapping function

$$L = H \times V$$

$$\varphi(x) = \frac{x}{1+x}$$



compress it by the global mapping function

$$L' = H \times V'$$

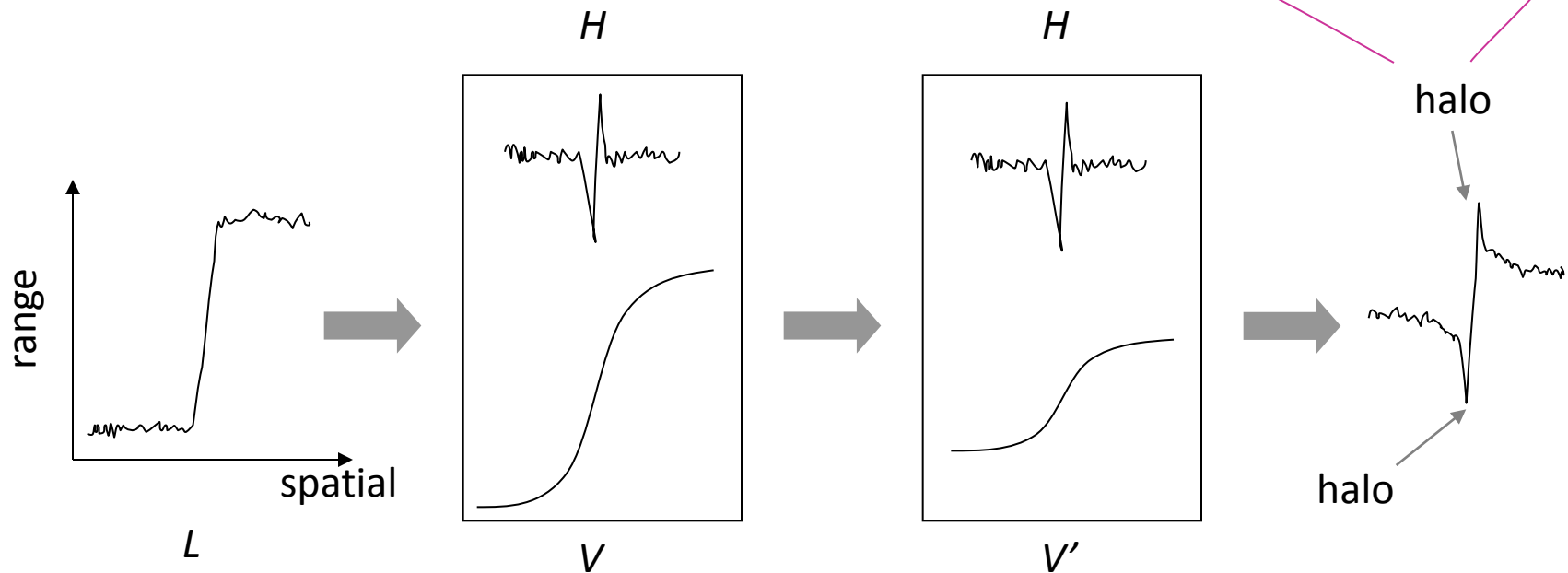
$$= \left(\frac{L}{V}\right) \times \left(\frac{V}{1+V}\right) = \boxed{\frac{L}{1+V}}$$

spatial varying

local mapping function

Halos

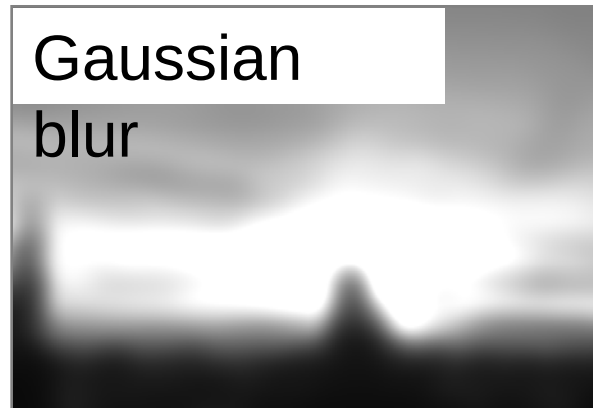
› Inverse contrasts/gradients





Preventing Halos

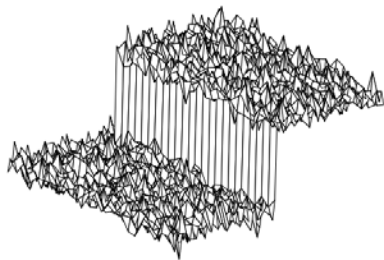
- › Need to construct a more appropriate local adaptation luminance V
- › Local averaging without blurring edges
 - › Multi-scale center-surround



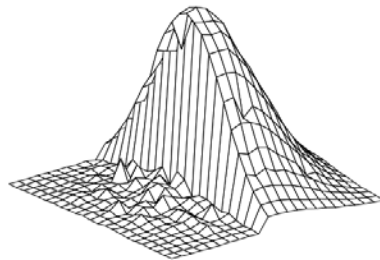
Bilateral Filtering

- › BILATERAL FILTERING FOR GRAY AND COLOR IMAGES, TOMASI AND MANDUCHI
 - › ICCV 1998

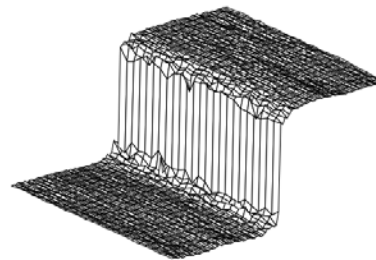
$$J(s) = \frac{1}{k(s)} \sum_{p \in \Omega} f(p - s) g(I_p^t - I_s^t) I_p^t$$



(a)



(b)



(c)

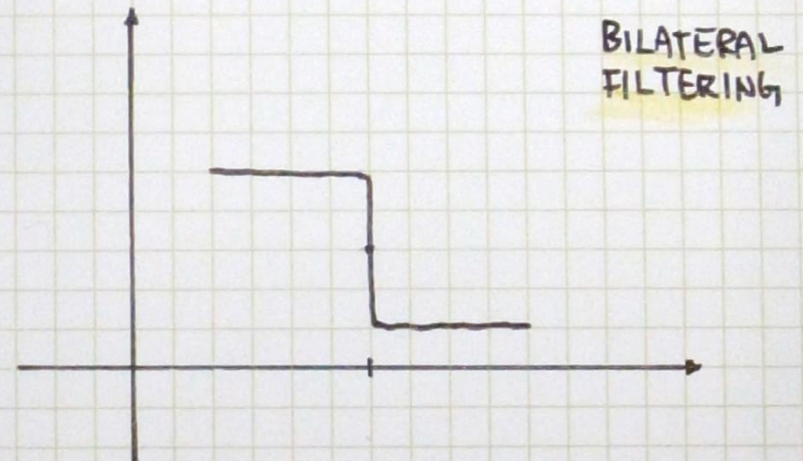
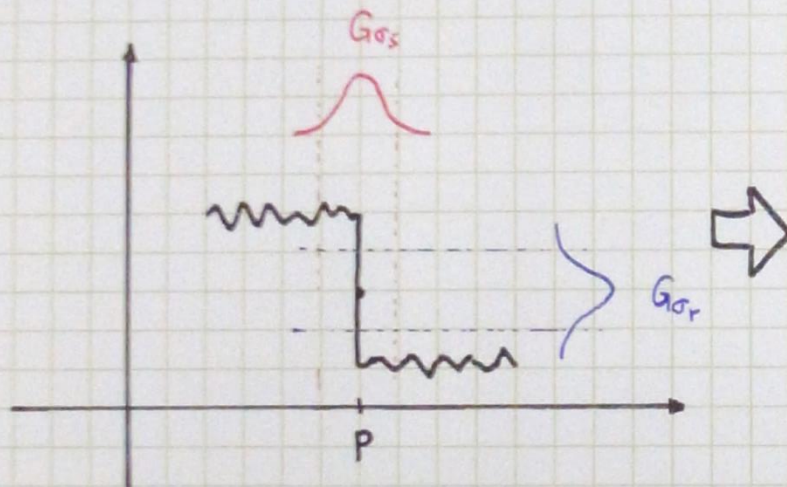
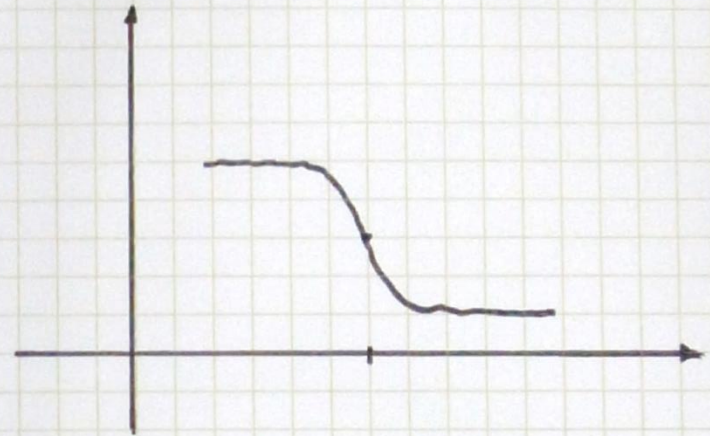
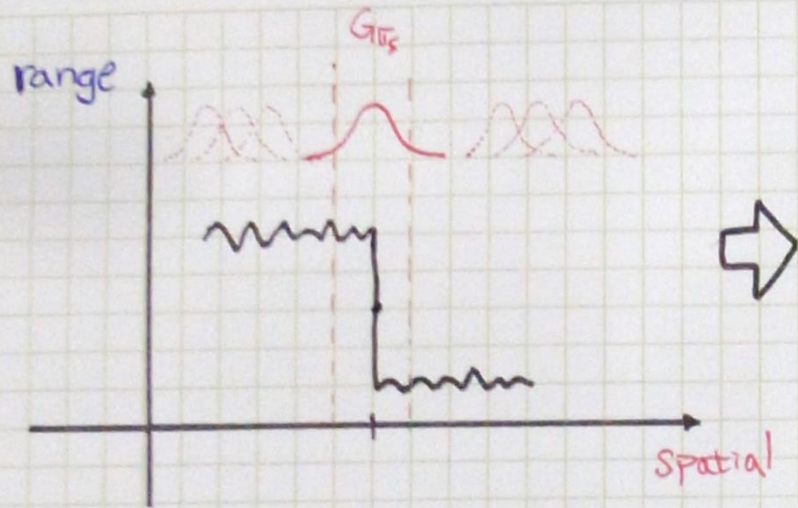


(a)

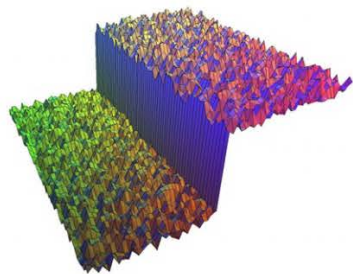


(b)

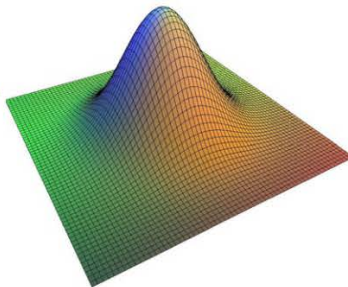
BILATERAL FILTERING & JOINT BILATERAL FILTERING



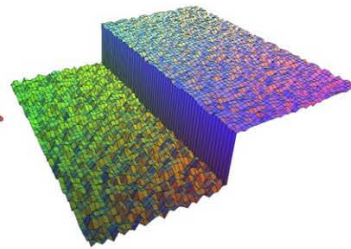
$$J(s) = \frac{1}{k(s)} \sum_{p \in \Omega} f(p - s) g(I_p^t - I_s^t) I_p^t$$



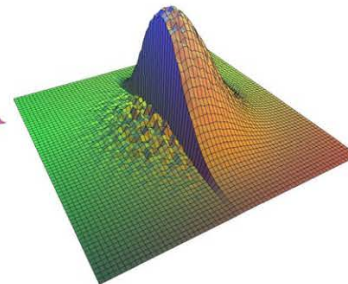
input



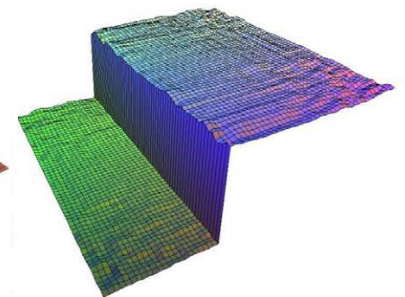
spatial kernel f



influence g in the intensity domain for the central pixel



weight $f \times g$ for the central pixel



output

-
- › *Fast Bilateral Filtering for the Display of High-Dynamic-Range Images*
 - › F. Durand's ppt slides

Acceleration

$$\begin{aligned} J_s^j &= \frac{1}{k^j(s)} \sum_{p \in \Omega} f(p-s) \ g(I_p - i^j) \ I_p \\ &= \frac{1}{k^j(s)} \sum_{p \in \Omega} f(p-s) \ H_p^j \end{aligned}$$

$$\begin{aligned} k^j(s) &= \sum_{p \in \Omega} f(p-s) \ g(I_p - i^j) \\ &= \sum_{p \in \Omega} f(p-s) \ G^j(p). \end{aligned}$$



PiecewiseBilateral

(Image I , spatial kernel f_{σ_s} , intensity influence g_{σ_r})

$J=0$ */* set the output to zero */*

for $j=0..NB_SEGMENTS$

$i^j = \min I + j \times (\max(I) - \min(I)) / NB_SEGMENTS$

$G^j = g_{\sigma_r}(I - i^j)$ */* evaluate g_{σ_r} at each pixel */*

$K^j = G^j \otimes f_{\sigma_s}$ */* normalization factor */*

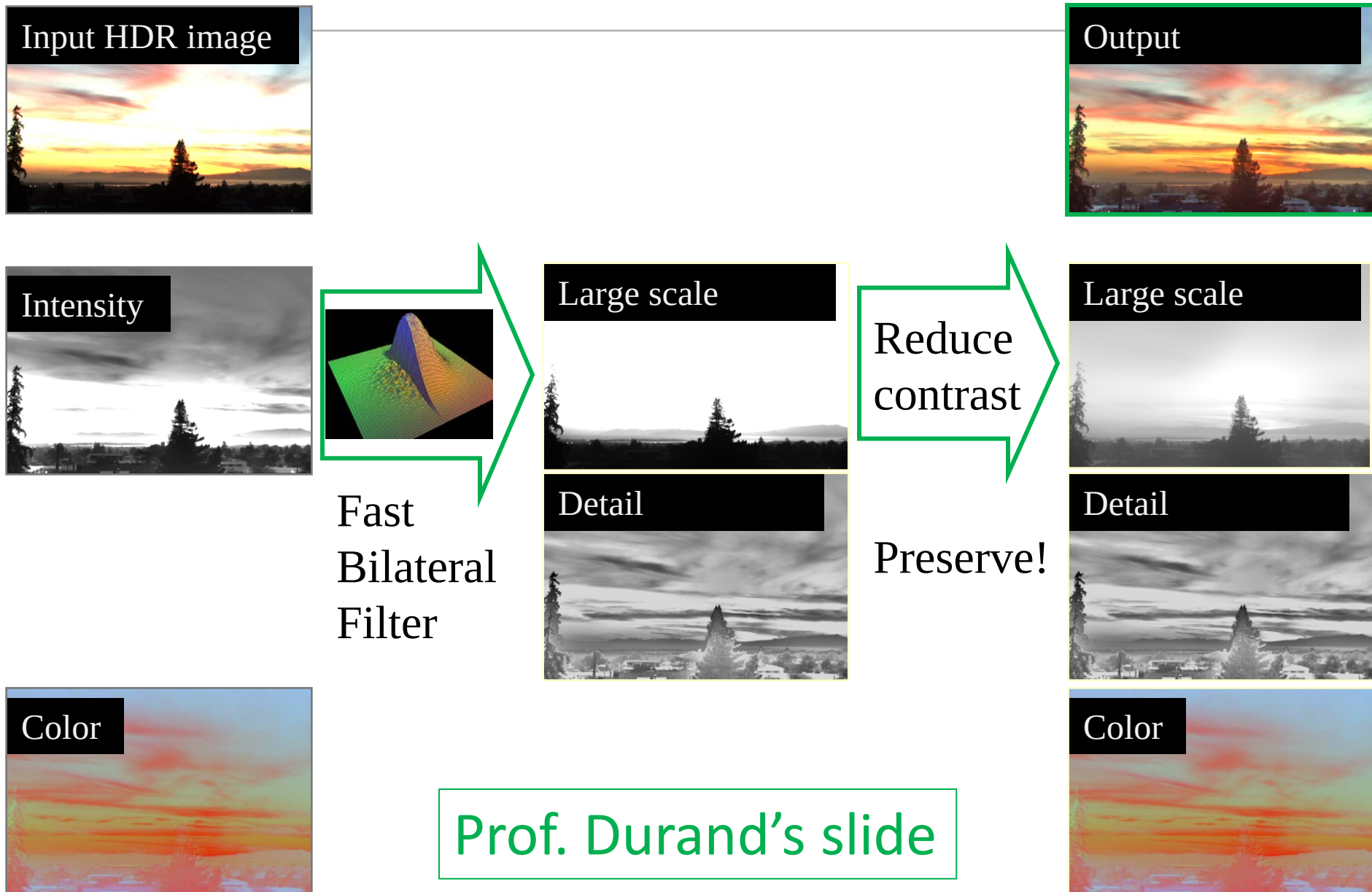
$H^j = G^j \times I$ */* compute H for each pixel */*

$H^{*j} = H^j \otimes f_{\sigma_s}$

$J^j = H^{*j} / K^j$ */* normalize */*

$J = J + J^j \times \text{InterpolationWeight}(I, i^j)$

Contrast reduction



Anisotropic Diffusion

› Heat propagation

$$\frac{\partial I}{\partial t} = \operatorname{div} [g(\|\nabla I\|) \nabla I] \quad \text{for edge stopping, } g = ?$$

› Perona & Malik

$$g_1(x) = \frac{1}{1 + \frac{x^2}{\sigma^2}} \quad g_2(x) = e^{-x^2/\sigma^2}$$

a large $\|\nabla I\|$ means a small g

› Discrete version

$$I_s^{t+1} = I_s^t + \frac{\lambda}{4} \sum_{p \in N_4(s)} g(I_p^t - I_s^t) (I_p^t - I_s^t)$$

Robust Anisotropic Diffusion

- › Robust to outliers

- › Black et al.

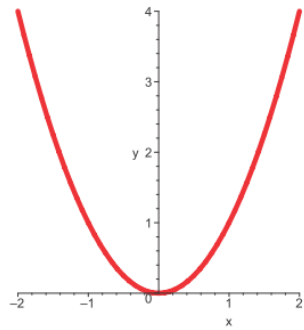
$$\text{minimize } \sum_{s \in \Omega} \sum_{p \in N_4(s)} \rho(I_p^t - I_s^t)$$

minimize error
over the whole image

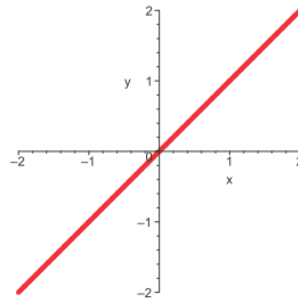
$$I_s^{t+1} = I_s^t + \frac{\lambda}{4} \sum_{p \in N_4(s)} \psi(I_p^t - I_s^t)$$

the influence $\psi(x) = \rho'(x)$

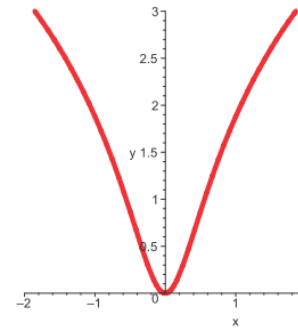
Perona & Malik $\psi(\nabla I) = g(\nabla I) \nabla I$



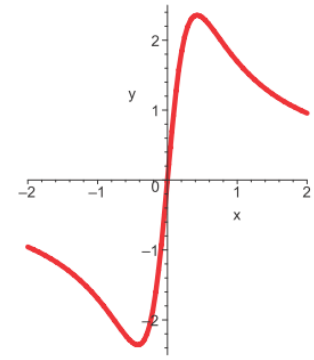
Least square $\rho(x)$



$\psi(x)$

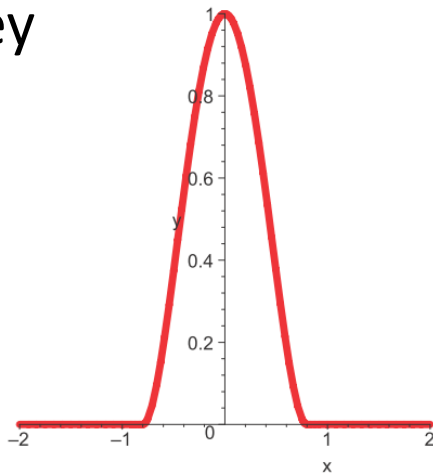


Lorentz $\rho(x)$

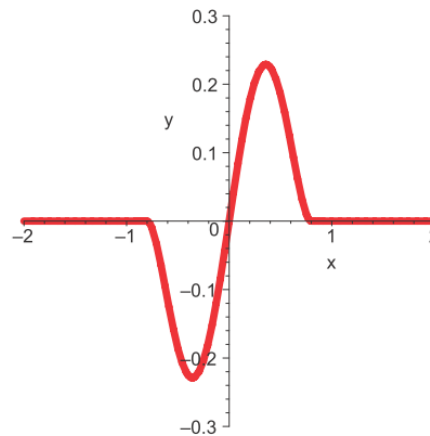


$\psi(x)$

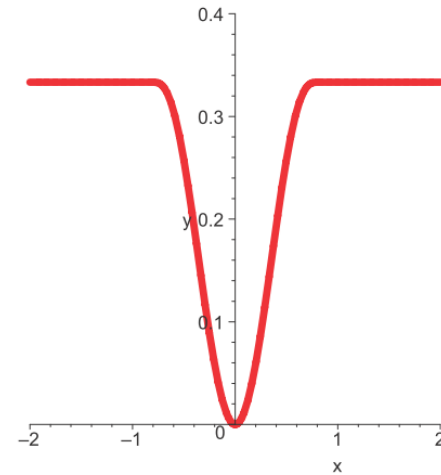
Tuckey



$g(x)$



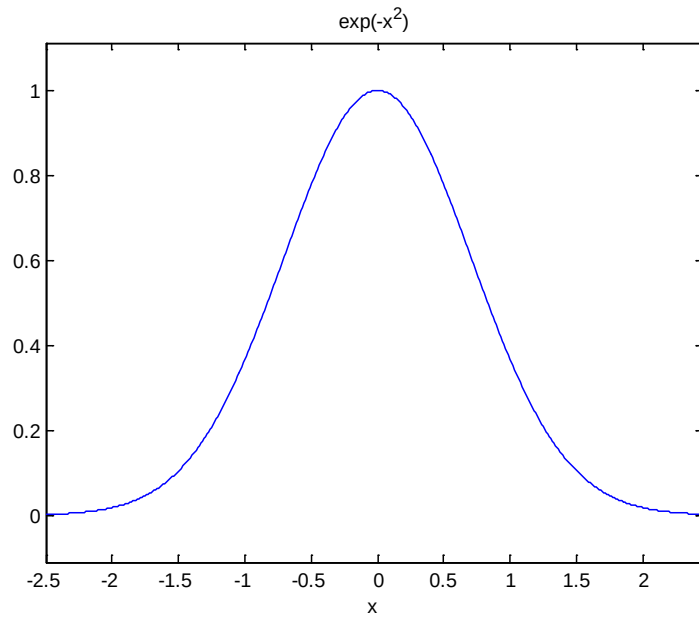
$\psi(x)$



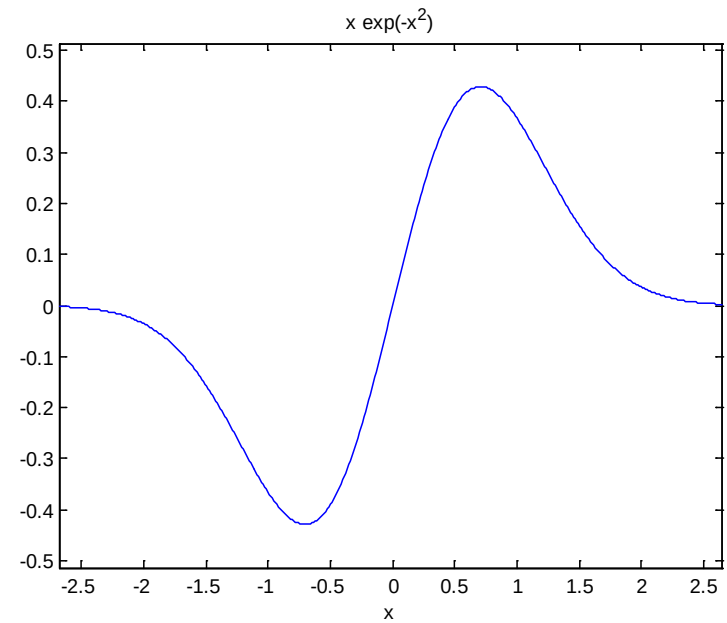
$\rho(x)$

Gaussian

$g(x)$



$\psi(x)$



$$\psi(x) = x g(x)$$

Huber	Lorentz
$g_{\sigma}(x) = \begin{cases} \frac{1}{\sigma} & x \leq \sigma \\ \frac{1}{ x }, & \text{otherwise} \end{cases}$ σ	$g_{\sigma}(x) = \frac{2}{2 + \frac{x^2}{\sigma^2}}$ $\sigma/\sqrt{2}$
Tukey	Gauss
$g_{\sigma}(x) = \begin{cases} \frac{1}{2}[1 - (x/\sigma)^2]^2 & x \leq \sigma \\ 0, & \text{otherwise} \end{cases}$ $\sigma * \sqrt{5}$	$g_{\sigma}(x) = e^{-\frac{x^2}{2\sigma^2}}$ σ

Extend the 0-Order Anisotropic Diffusion to a Larger Support

$$I_s^{t+1} = I_s^t + \frac{\lambda}{4} \sum_{p \in N_4(s)} g(I_p^t - I_s^t) (I_p^t - I_s^t)$$



$$I_s^{t+1} = I_s^t + \frac{\lambda}{4} \sum_{p \in \Omega} f(p - s) g(I_p^t - I_s^t) (I_p^t - I_s^t)$$

- › Energy preserving
 - › Symmetric

Bilateral Filtering

- › Non-iterative

$$J(s) = \frac{1}{k(s)} \sum_{p \in \Omega} f(p - s) g(I_p^t - I_s^t) I_p^t$$

- › Not energy preserving

$k(s)$ differs

Related Work on Bilateral Filtering

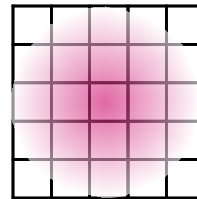
- › BILATERAL FILTERING: PAPERS, RESOURCES, APPLICATIONS, PARIS AND DURAND
- › CONSTANT TIME $O(1)$ BILATERAL FILTERING PORIKLI
 - › CVPR 2008
- › REAL-TIME $O(1)$ BILATERAL FILTERING, YANG, TAN AND AHUJA
 - › CVPR 2009
- › SVM FOR EDGE-PRESERVING FILTERING, YANG, WANG AND AHUJA
 - › CVPR 2010
- › Image Smoothing via L0 Gradient Minimization, XU, LU, XU, AND JIA
 - › SIGGRAPH Asia 2011.

Beyond Bilateral Filtering

- › Non-local averaging
 - › For denoising

$$NLu(\mathbf{x}) = \frac{1}{C(\mathbf{x})} \int_{\Omega} e^{-\frac{(G_{\rho} * |u(\mathbf{x} + \cdot) - u(\mathbf{y} + \cdot)|^2)(0)}{h^2}} u(\mathbf{y}) d\mathbf{y}$$

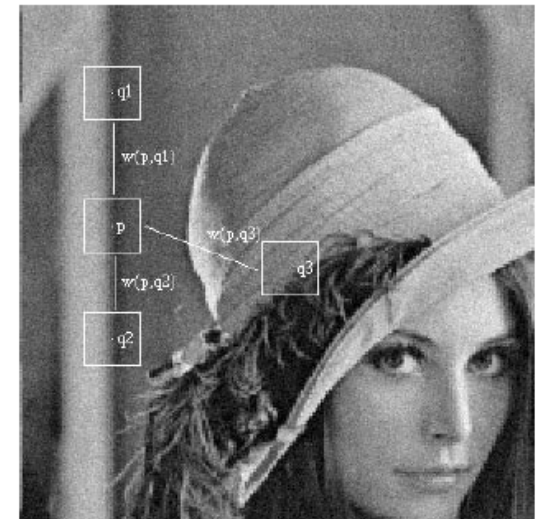
$$(G_{\rho} * |u(\mathbf{x} + \cdot) - u(\mathbf{y} + \cdot)|^2)(0) = \int_{\mathbb{R}^2} G_{\rho}(\mathbf{t}) |u(\mathbf{x} + \mathbf{t}) - u(\mathbf{y} + \mathbf{t})|^2 d\mathbf{t}.$$



- › <http://mw.cmla.ens-cachan.fr/megawave/demo/nlmeans/>
- › Buades, Coll, and Morel

- › Cross/joint bilateral filtering

self-similarity



Trilateral Filter [Choudhury&Tumblin]

$$G_{\theta}(\mathbf{x}) = \frac{1}{k_{\theta}(\mathbf{x})} \int_{-\infty}^{\infty} \nabla I_{in}(\mathbf{x} + \zeta) \frac{c(\zeta) s(\|\nabla I_{in}(\mathbf{x} + \zeta) - \nabla I_{in}(\mathbf{x})\|)}{d\zeta}$$

modified gradient

